Goethe-Universität Frankfurt
Institut für Mathematik
Sommersemester 2026
21. April 2026

Riemannsche Flächen
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Übungsblatt 1

Aufgabe 1

Let $X = V/\Lambda$ be a complex torus.

(a) Show that X admits a complex subtorus of dimension g' if and only if there exists a subgroup $\Lambda' \subset \Lambda$ of rank $2g'$ such that the image of the canonical map $\Lambda' \otimes \mathbb{R} \to V$ is a $\mathbb{C}$-subvector space of V.

(b) Conclude from a) that every complex torus admits at most countably many complex subtori.

(c) Give an example of a complex torus of dimension ≥ 2 not admitting any nontrivial complex subtorus.

Aufgabe 2

Let $X = V/\Lambda$ be a complex torus of dimension g. Show that

(a) there exist bases of V and Λ with respect to which the period matrix of X is of the form (Z, Id_g) with $Z \in M_g(\mathbb{C})$ and $\det \operatorname{Im} Z \neq 0$.
Hint: Can you get a base of V from Λ?

(b) $\det \begin{bmatrix} Z & Id_g \\ \overline{Z} & Id_g \end{bmatrix} = \det(2i \operatorname{Im} Z)$. Hint: Multiply by $\begin{bmatrix} Id_g & -Id_g \\ 0 & Id_g \end{bmatrix}$.

Aufgabe 3

(a) There is a bijection between the set of complex structures on the vector space $\mathbb{R}^{2g}$ and $\mathrm{GL}_{2g}(\mathbb{R})/\mathrm{GL}_g(\mathbb{C})$.
Hint: Prove that every complex structure J is conjugate to $J_0 = \begin{bmatrix} 0 & -Id_g \\ Id_g & 0 \end{bmatrix}$.

(b) This indices a bijection between the set of isomorphism classes of complex tori of dimension g and the set of orbits in $\mathrm{GL}_{2g}(\mathbb{R})/\mathrm{GL}_g(\mathbb{C})$ under the natural action of $\mathrm{GL}_{2g}(\mathbb{Z})$.

Abgabe bis Beginn der Übung um **14:00** am **Dienstag, den 28. April**.